On the Symmetry of Universal Finite-Size Scaling Functions in Anisotropic Systems

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In this work a symmetry of universal finite-size scaling functions under a certain anisotropic scale transformation is postulated. This transformation connects the properties of a finite two-dimensional system at criticality with generalized aspect ratio $\rho > 1$ to a system with $\rho < 1$. The symmetry is formulated within a finite-size scaling theory, and expressions for several universal amplitude ratios are derived. The predictions are confirmed within the exactly solvable weakly anisotropic two-dimensional Ising model and are checked within the two-dimensional dipolar in-plane Ising model using Monte Carlo simulations. This model shows a strongly anisotropic phase transition with different correlation length exponents $\nu_\parallel \neq \nu_\perp$ parallel and perpendicular to the spin axis.

The theory of universal finite-size scaling (UFSS) functions is a key concept in modern understanding of continuous phase transitions [1, 2, 3]. In particular, it is known that the UFSS functions of a rectangular two-dimensional (2D) system of size $L_\parallel \times L_\perp$ depend on the aspect ratio $L_\parallel/L_\perp$ [4]. For instance, in *isotropic* systems the scaling function at criticality $\bar{U}_\mathrm{c}$ of the Binder cumulant $U = 1 - \frac{1}{3}\langle m^4\rangle/\langle m^2\rangle^2$ [5], where $\langle m^n\rangle$ is the n-th moment of the order parameter, is known to be a universal function $\bar{U}_\mathrm{c}(L_\parallel/L_\perp)$ for a given boundary condition. This quantity has been investigated by several authors in the isotropic 2D Ising model with periodic boundary conditions [6, 7], while the influence of other boundary conditions on $\bar{U}_\mathrm{c}(L_\parallel/L_\perp)$ has recently been studied in Refs. [8, 9].

In *weakly anisotropic* systems, where the couplings are anisotropic ($J_\parallel \neq J_\perp$ in the 2D Ising case), the correlation length of the infinite system in direction $\mu = \parallel, \perp$ becomes anisotropic and scales like $\xi_\mu^{(\infty)}(t) \sim \hat{\xi}_\mu t^{-\nu}$ near criticality. ($t = (T - T_\mathrm{c})/T_\mathrm{c}$ is the reduced temperature and we assume $t > 0$ without loss of generality.) This leads to a correlation length amplitude ratio $\hat{\xi}_\parallel/\hat{\xi}_\perp$ different from unity. The UFSS functions then depend on this ratio, i. e. $\bar{U}_\mathrm{c} = \bar{U}_\mathrm{c}(L_\parallel/L_\perp, \hat{\xi}_\parallel/\hat{\xi}_\perp)$. However, isotropy can be restored asymptotically by an anisotropic scale transformation, where all lengths are rescaled with the corresponding correlation length amplitudes $\hat{\xi}_\mu$ [10, 11, 12]. Thus the UFSS functions depend on $L_\parallel/L_\perp$ and $\hat{\xi}_\parallel/\hat{\xi}_\perp$ only through the *reduced* aspect ratio $(L_\parallel/\hat{\xi}_\parallel)/(L_\perp/\hat{\xi}_\perp)$.

In *strongly anisotropic* systems both the amplitudes $\hat{\xi}_\mu$ as well as the correlation length exponents ν_μ are different and the correlation length in direction μ scales like

$$\xi_\mu^{(\infty)}(t) \sim \hat{\xi}_\mu t^{-\nu_\mu}. \tag{1}$$

Examples for strongly anisotropic phase transitions are Lifshitz points [13] as present in the anisotropic next nearest neighbor Ising (ANNNI) model [14, 15, 16], or the non-equilibrium phase transition in the driven lattice gas model [17, 18]. Furthermore, in dynamical systems one can identify the $\parallel$–direction with time and the $\perp$–direction(s) with space [19], which in most cases give strongly anisotropic behavior.

Using the same arguments as above we conclude that UFSS functions of strongly anisotropic systems depend on the *generalized* reduced aspect ratio (cf. [6])

$$\rho = L_\parallel L_\perp^{-\theta}/r_\xi, \quad \text{with} \quad r_\xi = \hat{\xi}_\parallel \hat{\xi}_\perp^{-\theta} \tag{2}$$

being the *generalized* correlation length amplitude ratio, and with the anisotropy exponent $\theta = \nu_\parallel/\nu_\perp$ [19]. Up to now no attempts have been made to describe the dependency of UFSS functions like $\bar{U}_\mathrm{c}(\rho)$ on the shape ρ of strongly anisotropic systems. In particular, it is not known if the anisotropy exponent θ can be determined from $\bar{U}_\mathrm{c}(\rho)$. This problem is addressed in this work.

Consider a 2D strongly anisotropic finite system with periodic boundary conditions. When the critical point of the infinite system is approached from temperatures $t > 0$, the correlation lengths ξ_μ in the different directions μ are limited by the direction in which $\xi_\mu^{(\infty)}$ from Eq. (1) reaches the system boundary first [4]. For a given volume $N = L_\parallel L_\perp$ we define an "optimal" shape $L_\parallel^\mathrm{opt} \times L_\perp^\mathrm{opt}$ at which both correlation lengths $\xi_\mu^{(\infty)}$ reaches the system boundary simultaneously, i. e.

$$L_\mu^\mathrm{opt} := \xi_\mu^{(\infty)}(t) \tag{3}$$

for some temperature $t > 0$ (Fig. 1a). We immediately find using Eqs. (1, 2) that the optimal shape obeys $\rho_\mathrm{opt} \equiv 1$ for all N, giving $L_\parallel^\mathrm{opt} = r_\xi (L_\perp^\mathrm{opt})^\theta$. A system of optimal shape should show the strongest critical fluctuations for a given volume N as the critical correlation volume $\xi_{\parallel,\mathrm{c}}\xi_{\perp,\mathrm{c}}$ spans the whole system.

At the optimal aspect ratio $\rho = 1$ the correlations are limited by both directions $\parallel$ and $\perp$ (Fig. 1a). If the system is enlarged by a factor $b > 1$ in the $\parallel$–direction (Fig. 1b), the correlation volume may relax into this direction but does not fill the whole system due to the limitation in $\perp$–direction. A similar situation with exchanged roles occurs if the system is enlarged by a factor $b > 1$ in the $\perp$–direction (Fig. 1c). We now *assume* that systems (b) and (c) are similar in the scaling region $L_\mu^\mathrm{opt} \to \infty$, i. e. that their correlation volumes are asymptotically equal.

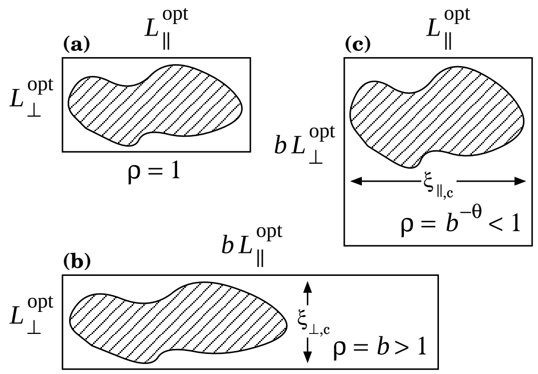

Figure 1: Three systems with different aspect ratio ρ (Eq. (2)) at criticality. In (a) the critical correlation volume $\xi_{\parallel,\mathrm{c}}\xi_{\perp,\mathrm{c}}$ (shaded area) spans the whole system, while in (b) and (c) correlations are limited by symmetric finite-size effects.

Hence we can formulate a *symmetry hypothesis*: Consider a system with periodic boundary conditions and optimal aspect ratio $\rho = 1$ at the critical point. If this system is enlarged by a factor $b > 1$ in $\parallel$–direction, it behaves asymptotically the same as if enlarged by *the same factor* b in $\perp$–direction.

To formulate this hypothesis within a finite-size scaling theory, we consider a 2D strongly anisotropic system of size $L_\parallel \times L_\perp$ which fulfills the generalized hyperscaling relation $2-\alpha = \nu_\parallel + \nu_\perp$ [6]. For our purpose it is sufficient to focus on the critical point. The universal finite-size scaling *ansatz* [1, 2, 3, 4, 6] for the singular part of the free energy density $f_\mathrm{c} = F_{\mathrm{s,c}}/(N k_\mathrm{B} T_\mathrm{c})$ reads [20]

$$f_\mathrm{c}(L_\parallel, L_\perp) \sim \frac{b_\parallel b_\perp}{N} Y_\mathrm{c}(b_\parallel, b_\perp) \tag{4}$$

with the scaling variables $b_\mu = \lambda^{\nu_\mu} L_\mu/\hat{\xi}_\mu$, where λ is a free scaling parameter. The scaling function Y_c is universal for a given boundary condition, all non-universal properties are contained in the metric factors $\hat{\xi}_\mu$. These metric factors occur due to the usual requirement that the relevant lengths are $L_\mu/\xi_\mu^{(\infty)}(t)$ near criticality, and cannot be absorbed into λ in contrast to isotropic systems. For the three systems in Fig. 1 we set $\lambda = (L_\mu^\mathrm{opt}/\hat{\xi}_\mu)^{-1/\nu_\mu}$ to get

$$f_\mathrm{c}(L_\parallel^\mathrm{opt}, L_\perp^\mathrm{opt}) \sim \frac{1}{N} Y_\mathrm{c}(1,1) \tag{5a}$$

$$f_\mathrm{c}(bL_\parallel^\mathrm{opt}, L_\perp^\mathrm{opt}) \sim \frac{b}{N} Y_\mathrm{c}(b,1) \tag{5b}$$

$$f_\mathrm{c}(L_\parallel^\mathrm{opt}, bL_\perp^\mathrm{opt}) \sim \frac{b}{N} Y_\mathrm{c}(1,b). \tag{5c}$$

The proposed symmetry hypothesis states that for $b > 1$ Eqs. (5b) and (5c) are asymptotically equal in the scaling region where L_μ^opt is large,

$$f_\mathrm{c}(bL_\parallel^\mathrm{opt}, L_\perp^\mathrm{opt}) \overset{b\geq 1}{\simeq} f_\mathrm{c}(L_\parallel^\mathrm{opt}, bL_\perp^\mathrm{opt}). \tag{6}$$

Hence the scaling function Y_c has the simple symmetry

$$Y_\mathrm{c}(b,1) \overset{b\geq 1}{=} Y_\mathrm{c}(1,b). \tag{7}$$

To rewrite Y_c as function of the generalized aspect ratio ρ (Eq. (2)) instead of the quantities b_μ, we set $b_\perp = 1$ in system (c) and get, as then $\lambda = (bL_\perp^\mathrm{opt}/\hat{\xi}_\perp)^{-1/\nu_\perp}$,

$$f_\mathrm{c}(L_\parallel^\mathrm{opt}, bL_\perp^\mathrm{opt}) \sim \frac{b^{-\theta}}{N} Y_\mathrm{c}(b^{-\theta}, 1). \tag{8}$$

Eqs. (5c) and (8) are identical and we conclude that $bY_\mathrm{c}(1,b) = b^{-\theta} Y_\mathrm{c}(b^{-\theta},1)$. At this point it is convenient to define the scaling function $\bar{Y}_\mathrm{c}(b) = bY_\mathrm{c}(b,1)$ which fulfills

$$f_\mathrm{c}(L_\parallel, L_\perp) \sim \frac{1}{N} \bar{Y}_\mathrm{c}(\rho). \tag{9}$$

For this scaling function the symmetry reads

$$\bar{Y}_\mathrm{c}(\rho) \overset{\rho\geq 1}{=} \bar{Y}_\mathrm{c}(\rho^{-\theta}). \tag{10}$$

We see from Eq. (9) that the critical free energy density f_c is a universal function of the reduced aspect ratio $\rho = L_\parallel L_\perp^{-\theta}/r_\xi$ without any non-universal prefactor, and that at criticality *all* system specific properties are contained in the non-universal ratio r_ξ from Eq. (2).

Ansatz Eq. (4) can also be made for the inverse spin-spin correlation length at criticality [20]

$$\xi_{\mu,\mathrm{c}}^{-1}(L_\parallel, L_\perp) \sim \frac{b_\mu}{L_\mu} X_{\mu,\mathrm{c}}(b_\parallel, b_\perp). \tag{11}$$

The proposed symmetry gives $X_{\mu,\mathrm{c}}(b,1) \overset{b\geq 1}{=} X_{\bar{\mu},\mathrm{c}}(1,b)$, where $\bar{\mu}$ denotes the direction perpendicular to μ. Hence the correlation volumes $\xi_{\parallel,\mathrm{c}}\xi_{\perp,\mathrm{c}}$ of system (b) and (c) in Fig. 1 are indeed equal as assumed above and become $\xi_{\parallel,\mathrm{c}}\xi_{\perp,\mathrm{c}} \sim \frac{N}{b} X_{\parallel,\mathrm{c}}^{-1}(b,1) X_{\perp,\mathrm{c}}^{-1}(b,1)$.

The correlation length amplitudes A_ξ^μ in cylindrical geometry ($b_\mu \to \infty$, $b_{\bar{\mu}} = 1$), which can be calculated exactly for many isotropic two-dimensional models within the theory of conformal invariance [21] generalize to the strongly anisotropic form [3]

$$A_\xi^\mu = \lim_{L_{\bar{\mu}}\to\infty} L_{\bar{\mu}}^{-\nu_\mu/\nu_{\bar{\mu}}} \lim_{L_\mu\to\infty} \xi_{\mu,\mathrm{c}}(L_\parallel, L_\perp). \tag{12}$$

Inserting Eq. (11) they become

$$A_\xi^\parallel = r_\xi X_{\parallel,\mathrm{c}}^{-1}(\infty,1), \quad A_\xi^\perp = r_\xi^{-1/\theta} X_{\perp,\mathrm{c}}^{-1}(1,\infty) \tag{13}$$

which shows that in general A_ξ^μ ist not universal. The symmetry hypothesis states that both limits of the scaling function $X_{\mu,\mathrm{c}}$ are equal. Denoting this universal limit $A_\xi := X_{\parallel,\mathrm{c}}^{-1}(\infty,1) = X_{\perp,\mathrm{c}}^{-1}(1,\infty)$ we obtain $A_\xi^\parallel = r_\xi A_\xi$ and $A_\xi^\perp = r_\xi^{-1/\theta} A_\xi$ as well as the amplitude relations

$$A_\xi^{1+\theta} = A_\xi^\parallel (A_\xi^\perp)^\theta, \qquad \frac{A_\xi^\parallel}{A_\xi^\perp} = r_\xi^{1+1/\theta}. \tag{14}$$

These predictions can be checked within the exactly solved weakly anisotropic 2D Ising model with different couplings $J_\parallel$ and $J_\perp$, where the paramagnetic correlation length reads $\xi_\mu^{(\infty)}(t) = (\log\coth(\beta J_\mu) - 2\beta J_{\bar\mu})^{-1}$ with $\beta = 1/k_\mathrm{B}T$ [22]. The amplitude ratio r_ξ at the critical point $\sinh(2\beta_\mathrm{c}J_\parallel)\sinh(2\beta_\mathrm{c}J_\perp) = 1$ [22] becomes $r_\xi = \sinh(2\beta_\mathrm{c}J_\parallel)$ [23]. On the other hand, the inverse correlation length amplitudes in cylinder geometry Eq. (12) has been calculated [24] to give $A_\xi^\mu = \frac{4}{\pi}\sinh(2\beta_\mathrm{c}J_\mu)$, which immediately yields Eqs. (13) if we insert the well known universal value $A_\xi = 4/\pi$ [21, 25]. The left relation of Eqs. (14) has already been derived for several weakly anisotropic models, where it simplifies to $A_\xi^2 = A_\xi^\parallel A_\xi^\perp$ [24, Eq. (7)].

To check the symmetry numerically in strongly anisotropic systems, we now focus on the Binder cumulant U. The scaling *ansatz* at criticality Eq. (4) becomes

$$U_\mathrm{c}(L_\parallel, L_\perp) \sim \frac{1}{b_\parallel b_\perp}\tilde U_\mathrm{c}(b_\parallel, b_\perp) = \bar U_\mathrm{c}(\rho) \qquad (15)$$

with the scaling function $\bar U_\mathrm{c}(b) = \tilde U_\mathrm{c}(b,1)/b$, and the calculation is completely analogous to the free energy case. The symmetry hypothesis for the cumulant scaling functions $\tilde U_\mathrm{c}$ and $\bar U_\mathrm{c}$ thus reads (cf. Eqs. (7,10))

$$\tilde U_\mathrm{c}(b,1) \overset{b\geq 1}{=} \tilde U_\mathrm{c}(1,b), \qquad \bar U_\mathrm{c}(\rho) \overset{\rho\geq 1}{=} \bar U_\mathrm{c}(\rho^{-\theta}). \qquad (16)$$

The generalization of the cumulant amplitude A_U^μ [5, 26] to strongly anisotropic systems is similar to Eq. (12) and gives

$$A_U^\mu = \lim_{L_{\bar\mu}\to\infty} L_{\bar\mu}^{-\nu_\mu/\nu_{\bar\mu}} \lim_{L_\mu\to\infty} L_\mu U_\mathrm{c}(L_\parallel, L_\perp). \qquad (17)$$

Inserting the scaling *ansatz* Eq. (15) we now find

$$A_U^\parallel = r_\xi \tilde U_\mathrm{c}(\infty,1), \qquad A_U^\perp = r_\xi^{-1/\theta}\tilde U_\mathrm{c}(1,\infty), \qquad (18)$$

which again are in general not universal. Using the symmetry hypothesis we can define $A_U := \tilde U_\mathrm{c}(\infty,1) = \tilde U_\mathrm{c}(1,\infty)$ and get $A_U^\parallel = r_\xi A_U$, $A_U^\perp = r_\xi^{-1/\theta}A_U$ as well as the identities (cf. Eqs. (14))

$$A_U^{1+\theta} = A_U^\parallel (A_U^\perp)^\theta, \qquad \frac{A_U^\parallel}{A_U^\perp} = r_\xi^{1+1/\theta}. \qquad (19)$$

The cumulant scaling function $\bar U_\mathrm{c}(\rho)$ must be extremal at $\rho = 1$ due to symmetry. Furthermore, as a deviation from the optimal aspect ratio $\rho = 1$ reduces the cumulant, it has a maximum at this point [6]. A sketch of $\bar U_\mathrm{c}(\rho)$ for an assumed anisotropy exponent $\theta = 2$ is depicted in Fig. 2. For $\rho > 1$ both $\bar U_\mathrm{c}(\rho)$ and $\bar U_\mathrm{c}(\rho' = \rho^{-\theta})$ collapse onto a single curve, reflecting the proposed symmetry. It is obvious from Fig. 2 that $\bar U_\mathrm{c}(\rho)$ (and thus also $\bar Y_\mathrm{c}(\rho)$ from Eq. (10)) can not be analytic at $\rho = 1$ in strongly anisotropic systems, as the two branches $\bar U_\mathrm{c}(\rho)$ and $\bar U_\mathrm{c}(\rho')$ identical for $\rho > 1$ fork at $\rho = 1$ [20]. On

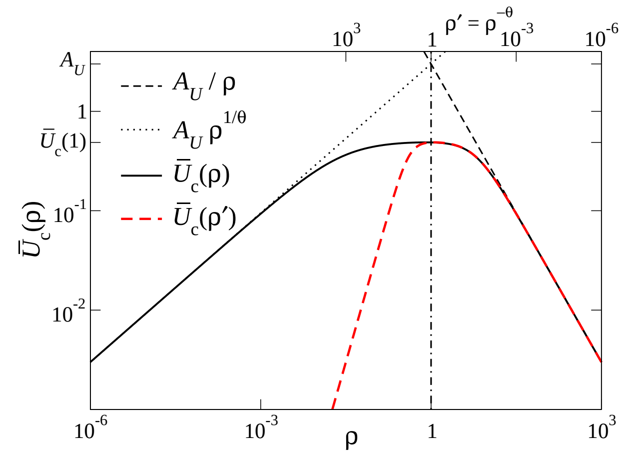

Figure 2: Sketch of critical cumulant scaling functions $\bar U_\mathrm{c}(\rho)$ and $\bar U_\mathrm{c}(\rho')$ with $\rho' = \rho^{-\theta}$ for assumed anisotropy exponent $\theta = 2$. We have $\bar U_\mathrm{c}(\rho \gg 1) \sim A_U/\rho$ and $\bar U_\mathrm{c}(\rho \ll 1) \sim A_U\rho^{1/\theta}$. For $\rho > 1$ $\bar U_\mathrm{c}(\rho)$ fulfills $\bar U_\mathrm{c}(\rho) = \bar U_\mathrm{c}(\rho')$.

the other hand, $\bar Y_\mathrm{c}(\rho)$ and $\bar U_\mathrm{c}(\rho)$ can be analytic at $\rho = 1$ if the anisotropy exponent $\theta = 1$, as in the case of the isotropic 2D Ising model [27, Eq. 3.37].

To check the symmetry hypothesis in a strongly anisotropic system, I performed Monte Carlo simulations of the two-dimensional dipolar in-plane Ising model [20]

$$\mathcal{H} = -\frac{J}{2}\sum_{\langle ij\rangle}\sigma_i\sigma_j + \frac{\omega}{2}\sum_{i\neq j}\frac{(r_{ij}^\perp)^2 - 2(r_{ij}^\parallel)^2}{|\vec r_{ij}|^5}\sigma_i\sigma_j \qquad (20)$$

with spin variables $\sigma = \pm 1$, ferromagnetic nearest neighbor exchange interaction $J > 0$, and dipole interaction $\omega > 0$. The distance $\vec r_{ij} = (r_{ij}^\parallel, r_{ij}^\perp)$ between spin σ_i and σ_j is decomposed into contributions parallel and perpendicular to the spin axis. In the simulations the Wolff cluster algorithm [28] for long range systems proposed by Luijten and Blöte [29] was used, modified to anisotropic interactions. In contrast to earlier work [30, 31] using renormalization group technics it is found that this model shows a strongly anisotropic phase transition. The details of the simulations will be published elsewhere [20].

After T_c was determined, systems with constant volume $N = L_\parallel L_\perp$ were simulated, which was chosen to have a large number of divisors in order to get many different aspect ratios (e.g. $N = 2^6 3^3 5^2 = 43200$ has 84 divisors). The resulting critical cumulant $U_\mathrm{c}(L_\parallel L_\perp^{-\theta})$ for two different volumes $N = 4320, 43200$ is depicted in the inset of Fig. 3. As expected, both curves have the same maximum value $\bar U_\mathrm{c}(1) = 0.555(5)$ at criticality. With variation of θ the curves are shifted horizontally and collapse for $\theta = 2.1(3)$, with maximum at $r_\xi = 0.415(40)$. To check the proposed symmetry we fold the left branch with $\rho < 1$ (open symbols) to the right and rescale the ρ-axis with θ. The resulting data collapse for $\rho > 1$ is shown in Fig. 3. This collapse and the additional condition that both curves must go to zero as A_U/ρ allows a

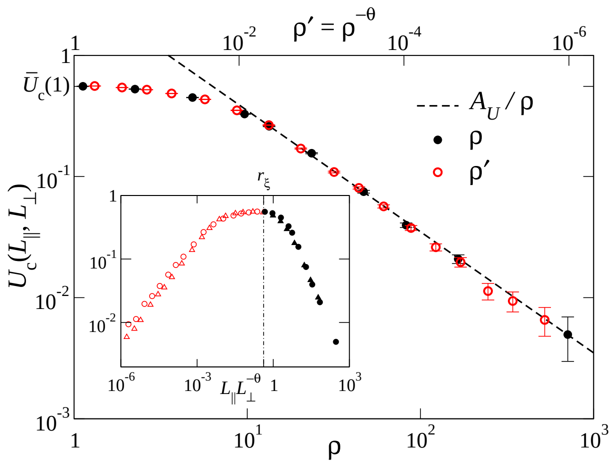

Figure 3: Cumulant $U_c(L_\parallel, L_\perp)$ of the dipolar in-plane Ising model (Eq. (20)) for dipole strength $\omega/J = 0.1$ and system size $N = 43200$ at the critical point $k_B T_c/J = 2.764(1)$. The data points collapse for $\rho > 1$ if we set $\theta = 2.1(3)$ and $r_\xi = 0.415(40)$, giving the universal amplitudes $\bar{U}_c(1) = 0.555(5)$ and $A_U = 3.5(2)$. The inset shows U_c as function of the unreduced generalized aspect ratio $L_\parallel L_\perp^{-\theta}$ for system size $N = 43200$ (circles) and $N = 4320$ (triangles).

precise determination of θ and r_ξ as well as of the universal amplitude $A_U = 3.5(2)$.

In conclusion, I postulate a symmetry of universal finite-size scaling functions under a certain anisotropic scale transformation and generalize the Privman-Fisher equations [1] to strongly anisotropic phase transitions on rectangular lattices at criticality. It turns out that for a given boundary condition the only relevant variable is the generalized reduced aspect ratio $\rho = L_\parallel L_\perp^\theta / r_\xi$ and that e.g. the free energy scaling function Eq. (9) obeys the symmetry $\bar{Y}_c(\rho) \stackrel{\rho \geq 1}{=} \bar{Y}_c(\rho^{-\theta})$. At criticality, the free energy density f_c, the inverse correlation lengths $\xi_{\mu,c}$, and the Binder cumulant U_c are universal functions of ρ, without a non-universal prefactor. All system specific properties are contained in the non-universal correlation length amplitude ratio r_ξ (Eq. (2)).

The generalization to higher dimensions is straightforward [20], an interesting application would be the precise determination of the exponent θ at the Lifshitz point of the three-dimensional ANNNI model [15, 16]. An open question is the validity of the proposed symmetry in non-equilibrium systems with appropriate boundary conditions, which recently have been shown to exhibit Privman-Fisher universality [3].

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